1. Optimización y Control Óptimo

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Índice

Convergencia de soluciones para sistemas de segundo orden de familias de operadores cocoercivos

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Resumen

Sea $\mathcal{H}$ un espacio *Hilbert*. Dado $A : \mathcal{H} \to 2^{\mathcal{H}}$ un operador maximal montono , $\alpha(t) > 0 \in \mathcal{C}^1(\mathcal{H})$ una funcin y $\gamma \in \mathbb{R}$ una constante, se estudian los sistemas diferenciales de segundo orden

$$\ddot{u}(t) + \gamma \dot{u}(t) + \alpha(t) A_{\alpha(t)} u(t) = 0, \quad t > t_0 \tag{1}$$

donde $A_{\alpha(t)}$ corresponde a la regularizacin de *Yosida* de A con parmetro $\alpha(t)$. Se mostrar que, bajo condiciones adecuadas para $\alpha(t)$ y γ, las soluciones $u(t)$ de dicho sistema convergen dbilmente a un elemento del conjunto

$$A^{-1}\{0\} = \bigcap_{t>t_0} \left(\alpha(t) A_{\alpha(t)}\right)^{-1} \{0\} \tag{2}$$

Este resultado es similar al probado por *lvarez* en [1]. Se presentar una formulacin discreta del sistema anterior dada por

$$\frac{u_{k+1} - 2u_k + u_{k-1}}{s^2} + \gamma \frac{u_{k+1} - u_k}{s} + \alpha_k A_{\alpha_k} u_{k+1} = 0$$

con $\{\alpha_k\}$ una sucesin positiva con condiciones similares al caso continuo. A partir de est, se propone el uso de un algoritmo RIPA[2] para obtener una secuencia minimizadora de forma eficiente. Finalmente mostraremos experimentos computacionales que motiven el uso de estas formulaciones y algoritmos.

Trabajo realizado en conjunto con:
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A Projected primal-dual splitting for solving constrained monotone inclusions

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Resumen

In this work, we provide a splitting algorithm for solving the problem of finding a point in the intersection of a closed convex subset X of a real Hilbert space and the set of zeros of a monotone operator. We assume that this operator can be decomposed in $A + L * BL + C$, where A and B are maximally monotone, C is cocoercive, and L is a linear operator with adjoint L^*. In the case when X is the whole space, the problem can be solved via the method proposed in [3] and [2], the latter in the context of optimization problems. In the context of constrained convex optimization, the previous algorithms force feasibility on primal iterates via Lagrange multiplier updates, which usually leads to slow algorithms whose primal iterates do not satisfy any of the constraints. In this work we propose a method that generalizes the algorithms in [3] and [2] by adding an appropriate projection on X, which, in the constrained optimization context, can be defined via a selection of the constraints onto which the projection can be computed. In this framework, we generate a primal sequence satisfying those constraints and obtaining improvement on the efficiency of the method, which is justified via numerical experiences. Finally, in the case when A or the inverse of B is strongly monotone, we provide an accelerated version of the algorithm ensuring a rate of convergence of $O(1/k^2)$. This version generalizes the accelerated method provided in [1].

Joint work with:
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Double regularization methods for robust feature selection and SVM classification via DC programming

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Resumen

In this work, two novel formulations for embedded feature selection are presented. A second-order cone programming approach for Support Vector Machines is extended by adding a second regularizer to encourage feature elimination. The one- and the zero-norm penalties are used in combination with the Tikhonov regularization under a robust setting designed to correctly classify instances, up to a predefined error rate, even for the worst data distribution. The use of the zero norm leads to a nonconvex formulation, which is solved by using Difference of Convex (DC) functions, extending DC programming to second-order cones. Experiments on high-dimensional microarray datasets were performed, and the best performance was obtained with our approaches compared with well-known feature selection methods for Support Vector Machines.

Joint work with:
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Asymptotic equivalence of evolution equations governed by monotone operators with some structure and their discretizations

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Resumen

In this work, we study discrete aproximations of the evolution equation

$$-x'(t) = Bx(t), \;\; t \; > \; 0, \; x(0) = x_0 \in \mathcal{H}, \tag{1}$$

by means of Euler iterations, and B a cocoercive operator on the Hilbert space $\mathcal{H}$. On the other hand, we give precise estimations for the distance between iterates of independently generated Euler sequences, and use them to obtain bounds for the distance between the state, given by the continuous-time trajectory, and the discrete approximations obtained by the Euler iterations. We establish the asymptotic equivalence between the continuous-and discrete-time systems, under a sharp hypothesis on the step sizes. The concept of asymptotic equivalence was introduced by Passty in [1], it means that continuous and discrete trajectories of evolution equation are convergent or divergents at the same time. Of particular interest is the case where $B = \nabla f$, with $f : \mathcal{H} \to \mathbb{R} \cup \{+\infty\}$ a convex and differentiable function, and ∇f is Lipschitz-continuous. According with Baillon-Haddad Theorem (see, for instance, [4, Theorem 3.13]), ∇f is a cocoercive operator and using some techniques of [3], we show the asymptotic equivalence between continuous trajectory of (1) and the iterates generated conveniently by gradient algorithm. In this case, the step sizes are chosen in a more general way. Asymptotic equivalence in this case allows us to construct a family of smooth functions for which the trajectories/sequences generated by first order methods converges weakly but not strongly, extending the counterexample of Baillon [2]. On the other hand, we include a few guidelines to address the problem in smooth Banach spaces. Finally, for the differential inclusion

$$-x'(t) \in (A + B)x(t), \; c.t.p \;\; t \; > \; 0, \; x(0) = x_0 \in \overline{D(A)}, \tag{2}$$

with $A : \mathcal{H} \to 2^{\mathcal{H}}$ is maximal monotone and B as above, we obtain estimations for the distance between iterates of independently generated forward-backward sequences. Furthermore, we present asymptotic equivalence between continuous-time trajectory associated to (2) and discrete trajectories obtained by forward-backward iterations.

Joint work with:
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Gradient flows and determination of convexity

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Resumen

We disclose an interesting connection between the gradient flow of a $\mathcal{C}^2$-smooth function ψ and strongly evanescent orbits of the second order gradient system defined by the square-norm of $\nabla\psi$, under an adequate convexity assumption. As a consequence, we obtain the following surprising result for two $\mathcal{C}^2$, convex and bounded from below functions ψ_1, ψ_2 (see [1]):
if $||\nabla\psi_1|| = ||\nabla\psi_2||$, then $\psi_1 = \psi_2 + k$, for some $k \in \mathbb{R}$.

Joint work with:
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Extended Euler-Lagrange and Hamiltonian Conditions in Optimal Control of Sweeping Processes with Controlled Moving Sets

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Resumen

This paper concerns optimal control problems for a class of sweeping processes governed by discontinuous unbounded differential inclusions that are described via normal cone mappings to controlled moving sets. Largely motivated by applications to hysteresis, we consider a general setting where moving sets are given as inverse images of closed subsets of finite-dimensional spaces under nonlinear differentiable mappings dependent on both state and control variables. Developing the method of discrete approximations and employing generalized differential tools of first-order and second-order variational analysis allow us to derive nondegenerated necessary optimality conditions for such problems in extended Euler-Lagrange and Hamiltonian forms involving the Hamiltonian maximization. The latter conditions of the Pontryagin Maximum Principle type are the first in the literature for optimal control of sweeping processes with control-dependent moving sets.

Joint work with:
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Self-dual approximations to fully convex impulsive systems

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Resumen

Fully convex optimal control problems contain a Lagrangian that is jointly convex in the state and velocity variables. Problems of this kind have been widely investigated by Rockafellar and collaborators if the Lagrangian is coercive and without state constraints. A lack of coercivity implies the dual has nontrivial state constraints, and vice versa (that is, they are dual concepts in convex analysis). We consider a framework using Goebel's self-dualizing technique that approximates both the primal and dual problem simultaneously and maintains the duality relationship. Previous results are applicable to the approximations, and we investigate the limiting behavior as the approximations approach the original problem. A specific example is worked out in detail and some extensions are presented.

Joint work with:
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Unbounded State Dependent Sweeping Processes with Perturbation in Uniformly Convex and q-Uniformly Smooth Banach Spaces

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Resumen

In this paper, the existence of solutions for a class of first and second order unbounded state-dependent sweeping processes with perturbation in uniformly convex and q-uniformly smooth Banach spaces are analyzed by using a discretization method. The sweeping process is a particular differential inclusion with a normal cone to a moving set and is of a great interest in many concrete applications. The boundedness of the moving set, which plays a crucial role for the existence of solutions in many works in the literature, is not necessary in the present paper. The compactness assumption on the moving set is also improved.

Joint work with:
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An interior algorithm for nonlinear conic programs: Application to classification problems

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Resumen

We present a new feasible direction algorithm for solving smooth nonlinear second-order cone programming problems. Given an interior point to the feasible set defined by the conic constraints, the proposed approach computes a feasible and descent direction, for the objective function, by solving two linear systems which result of applying the Newton method to the Karush-Kuhn-Tucker conditions of the nonlinear conic problem. Then, a line search along the search direction finds a new feasible point that has a lower value of the objective function. Repeating this process, the algorithm generates a feasible sequence with monotone decreasing values of the objective function. Under mild assumptions, we prove the global convergence of the algorithm. Numerical results are presented. Finally, we present new support vector classification formulations and then, we apply our approach for solving these models.

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Convergence notions for interval functions and their applications to interval optimization problems

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Resumen

We employ the variational convergence notion for vector functions defined in [2] and studied further in [3] to study the stability of interval optimization problems. We focus on set-type solutions of these problems that are defined by means of the Kulisch-Miranker order between intervals (see [1]). We describe the behavior under perturbations of the solution, level and colevel sets. We show that the coercivity and coercive existence conditions for these problems obtained in [1] are preserved locally within certain classes of functions. We compare the variational convergence with other convergence notions.

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Subdifferential characterization of probability functions under Gaussian distribution

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Resumen

This work provides formulae for the subdifferential of the probability function

$$\varphi(x) = \mathbb{P}(g(x,\xi) \leq 0),$$

where $(\Omega, \Sigma, \mathbb{P})$ is a probability space, ξ is an m-dimensional gaussian random vector, $g : X \times \mathbb{R}^m \to \mathbb{R}$ is locally Lipschitz and convex in the second variable and X is a separable reflexive Banach space. Applications for this class of functions can be found in water management, telecommunications, electricity network expansion, mineral blending, chemical engineering, etc, where the constraint $\mathbb{P}(g(x,\xi) \leq 0) \geq p$ expresses that a decision vector x is feasible if and only if the random inequality $g(x,\xi) \leq 0$ is satisfied with probability at least p. In [1, 2] the authors have provided criteria for the differentiability and Lipschitz continuity of the function φ together with formulas for the gradient and subgradients under the hypotheses that X is finite dimensional, g is the maximum of a finite number of continuously differentiable functions g_j and using some growth condition for the gradient $\nabla_x g(x,z)$ in a neighborhood of the point of interest $\bar{x}$, or assuming that the set $\{z \in \mathbb{R}^m : g(\bar{x}, z) \leq 0\}$ is bounded. The aim of this work is to extend the previous results to infinite dimensional spaces X, using a weaker growth condition and assuming local Lipschitz continuity of g only, even when the probabilistic function φ could be non-Lipschitz.

Joint work with:
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Inertial proximal algorithms for maximally monotone operators

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Resumen

We present a Regularized Inertial Proximal Algorithm to solve convex optimization problems and variational inequalities. It is obtained by means of a convenient finite-difference discretization of a second-order differential equation with vanishing damping, governed by the Yosida regularization of a maximally monotone operator with time-varying index. These systems are the counterpart to accelerated forward?backward algorithms in the context of maximally monotone operators. A simple example illustrates the behavior of these systems compared with some of their close relatives.

Joint work with:

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Stability Analysis for Parameterized Conic Programs

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Resumen

In this talk we first visit several results characterizing well-known stability properties (such as Aubin property, isolated calmness, etc.) for critical maps of parameterized conic programs. These characterizations are typically carried out via the computation of second order generalized derivatives, and we need the constraint set is defined over a convex cone satisfying a reducibility assumption and is (strongly) qualified. Then, we present an ongoing work, which aim is to prove/extend those results under weaker qualification constraints conditions, establishing some connections between second order derivatives and well-known conic tools, such as the sigma term. Our approach covers seminal examples such as (nonlinear) SDP and SOCP.

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Boundary of maximal monotone operators values

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Resumen

We characterize in Hilbert spaces the boundary of the values of maximal monotone operators, by means only of the values at nearby points, which are close enough to the reference point but distinct of it. This allows to write the values of such operators using finite convex combinations of the values at at most two nearby points. We also provide similar characterizations for the normal cone to prox-regular sets.

Joint work with:
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